LCM & HCF is an important topic asked under the Arithmetic section in Mathematics. It is asked in various Defence Exams such as CDS, AFCAT, Air Force Group X & Y etc.

We will continue this topic in various parts. We will discuss the basic questions that are usually asked in this topic and how you can solve them using the normal as well as Shortcut approach.

Tips & Tricks to solve LCM & HCF

Introduction of HCF

HCF stands for **'Highest Common Factor'**. To put simply, HCF of two (or more) numbers is the highest number that divides each of the numbers completely. For example in case of two numbers, 18 and 24, '6' is their HCF. Why? Because no other number greater than **'6' divides 18 and 24 completely.**

There is a rule that '**any number that divides each of the two numbers**, **also divides their sum, their difference and the sum and difference of any multiples** of two such numbers. HCF can be calculated using the following ways:

Prime Factorization: In this method, all the numbers are broken down to their prime factors. And, then common prime factors are multiplied to obtain HCF. To find the HCF of 1365, 1560 and 1755:

Note: Prime factors are the numbers which divide the number completely and are themselves prime numbers. Prime numbers are numbers that don't come in the table of any number but 1. Like 11 is a prime number.

Let's find prime factors of 1365, 1560 and 1755:

1365: **5**×3×7×13

1560: **5**×3×2×2×2×13

1755: **5**×3×3×3×13

We can see the common Prime Factors which have an equal number of frequency in each of the numbers are: 5, 3 and 13.

To get HCF, we'll multiply these numbers. So, **HCF** = 5×3×13 = 195

So, 195 is the largest number that completely divides 1365, 1560 and 1755.

Division:

a) If we've to find **HCF** of two numbers: **Divide greater number by smaller number**. Then divide the smaller number by remainder. Then, divide the remainder from last step by next remainder. Continue this cycle till remainder = 0. Last reminder to zero is our **HCF.**

To find HCF of 1365, 1560 and 1755.

First, we'll find the HCF of 1365 and 1560 using the method described above.

Step I: 1560 ÷ 1365 ⇒ Remainder = 195

Step II: 1365 ÷ 195 ⇒ Remainder = 0

⇒ HCF of 1365 and 1560 = 195

Step III: 1755 ÷ 195 ⇒ Remainder = 0

So, HCF of 1365, 1560 and 1755 = 195

Other types of questions based on HCF:

Question: Find the **greatest number** which is such that when 76, 151 and 226 are divided by it, the remainders are all alike. Also, find the common remainder.

Let the common remainder be 'k'

So, 76 – k, 151 – k and 226 – k are all divisible by that common number or HCF.

We know that if a number divides some numbers then it also divides their differences.

So, (151 – k) – (76 – k), (226 – k) – (151 – k) and (226 – k) – (76 – k) are also divisible by the HCF.

⇒ HCF divides 75, 75 and 150 also.

Therefore, the highest number that divides 75, 75 and 150 is their HCF

And, HCF of 75, 75 and 150 = 75

So, 75 divides 76 – k, 151 – k and 226 – k.

We can see on dividing 76 by 75 we get a remainder of 1 and 151÷75 gives the remainder of 1.

So, k = 1

Question: The **product of two** numbers is 7168 and their HCF is 16; find the numbers.

Here, the two numbers must have **HCF** as their factors so two numbers can be 16a and 16b where 'a' and 'b' are prime factors of each number respectively.

Now, $16a \times 16b = 7168$

$\Rightarrow ab = 28$

If a=1 then b =28 ⇒ Numbers are 16 and 448

If a = 4 then b = 7 ⇒ Numbers are 64 and 112

If a = 2 then b = 14 ⇒ But 2 and 14 have '2' in common among each other and we know that factors other than HCF don't have anything in common among each other so it's not possible.

So, the possible numbers are **(16, 448) or (64, 112).**

Question: In a school, 391 boys and 323 girls have been divided into the largest possible equal classes so that each class of boys numbers the same as each class of girls. What is the number of classes?

Number of classes = HCF of 391 and 323 = 17

Introduction of LCM

LCM stands for **'Least Common Multiple'**. To put simply, LCM of numbers is the least number which is completely divisible by each of the numbers. For example, 15 is divisible by 3 and 5.

Let's see different methods for calculating LCM:

Prime Factorization

Here, we multiply the prime factors with the highest powers to get the LCM as shown below:

To get LCM of 18, 24, 60 and 150, we write down the prime factors with highest powers.

$18 = 3\times3\times2 = 3^2\times2^1$

$24 = 3\times2\times2\times2 = 3^1\times2^3$

$60 = 3\times2\times2\times5 = 3^1\times2^2\times5^1$

$150 = 3\times2\times5\times5 = 3^1\times2^1\times5^2$

Here, the prime factors that occur in these numbers are 3, 2 and 5 and their highest powers are 3^2, 2^3 and 5^2

⇒ LCM = $3^2 \times 2^3 \times 5^2 = 1800$

Basic Line division:

Here, we write numbers on **RHS and on LHS** we write a prime factor. We divide each of the numbers by the prime factor on **LHS that divide different numbers**. And, then we write the quotient, we get

when previous **prime number** completely divides numbers on **RHS, in the next line**. Now again we write a prime number and repeat the process until we get all the quotients as 1.

2	18, 24, 60, 150
2	9, 12, 30, 75
2	9, 6, 15, 75
3	9, 3, 15, 75
3	3, 1, 5, 25
5	1, 1, 5, 25
5	1,1, 1, 5
	1, 1, 1, 1

Here, the LCM is a multiplication of all prime numbers on left

⇒ **LCM** = $2^3 \times 3^2 \times 5^2 = 1800$

Other types of questions based on LCM

Question: What least number must be subtracted from 1936 so that the remainder when divided by 9, 10, 15 will leave in each case the same remainder 7?

Step I: Find LCM of 9, 10 and 15 which is 90.

Step II: Divide 1936 by 90 ⇒ We get remainder as 46

Step III: From this remainder, we will subtract '7' ⇒ 46 – 7 = 39

So, the required number is 39.

Question: What is the least multiple of 7, which when divided by 2, 3, 4, 5 and 6 leaves the remainder 1, 2, 3, 4, 5 respectively?

Step I: Find LCM of 2, 3, 4, 5 and 6 which is 60

Step II: Number is 60k where k is a positive integer

Step III: Now, we'll subtract respective remainders from numbers as 2 leaves remainder of 1 so 2 – 1 = 1; similarly, 3 – 2 = 1, 4 – 3 = 1, 5 – 4 = 1, 6 – 5 = 1

As remainder in each case is less than divisor by 1

⇒ The required number is (60k – 1)

Step IV: Since the number is multiple of 7 so we'll find the least value that will make the number divisible by 7.

When k = 1

60k – 1 = 60×1 – 1 = 59 ⇒ Not divisible by 7

When k = 2

60k – 1 = 60×2 – 1 = 119 ⇒ Divisible by 7.

Question: The traffic lights at three different road crossing change after every 48 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 8:20 hrs then at what time will they change again simultaneously?

LCM of 48, 72, 108 = 432

The traffic lights will change simultaneously after 432 seconds or 7m 12sec

∴ they will change simultaneously at 8:20:00 + 00:07:12 = 8:27:12 hrs.

Mixed questions based on HCF & LCM:

The product of two numbers = HCF × LCM (of both numbers)

Fractions

$$\text{LCM of fraction} = \frac{\text{LCM of numerator}}{\text{HCF of Denominator}}$$

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$$\text{To find HCF of } \frac{54}{9}, \frac{39}{17} \text{ and } \frac{36}{51}$$

$$\text{Thus fractions are } 6, \frac{60}{17} \text{ and } \frac{12}{17}$$

$$\text{HCF of fraction} = \frac{\text{HCF of 6,60 and 36}}{\text{LCM of 1,17 and 17}} = 6/17$$

Question: Three men start to travel around a circular path of 11 km. Their speeds are 4, 5.5 and 8 kmph respectively. When will they meet at the starting point?

Sol. Time taken by them to complete one revolution will be (11/4), (11/5.5), (11/8) hrs respectively

$$\text{LCM of } \frac{11}{4}, 2, \text{ and } \frac{11}{8} = \frac{\text{LCM of 11, 2 and 11}}{\text{HCF of 4,1 and 8}}$$

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